Effective field approach to the Ising film in a transverse field

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Abstract

We study the phase transitions of the spin-$\frac{1}{2}$ Ising film in a transverse field within the framework of the effective field theory. We evaluate the critical temperature of the film as a function of the exchange interactions, the transverse field and the film thickness. We find that, if the ratio of the surface exchange interactions to the bulk ones $R = J_{\mathrm{s}}/J$ is smaller than a critical value R_{c}, the critical temperature T_{c}/J of the film is smaller than the bulk critical temperature $T_{\mathrm{c}}^{\mathrm{B}}/J$ and approaches $T_{\mathrm{c}}^{\mathrm{B}}/J$ as R increases further. On the other hand, if $R > R_{\mathrm{c}}$, T_{c}/J is larger than both the bulk $T_{\mathrm{c}}^{\mathrm{B}}/J$ and the surface $T_{\mathrm{c}}^{\mathrm{S}}/J$ critical temperatures of the corresponding semi-infinite system, and approaches $T_{\mathrm{c}}^{\mathrm{S}}/J$ as R increases further.

P.A.C.S.: 75.70.Ak, 75.40.Cx, 05.50.+q

Key words: Magnetic layers, effective field, surface transition.

1 Introduction

During the last years much effort has been directed towards the study of critical phenomena in various magnetic layered structures, ultrathin films and superlattices [1–4]. The basic theoretical problem is the examination of the

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magnetic excitation and the phase transitions in these systems. Of these, magnetic films are very important both from the theoretical and the experimental standpoints [5,6], and can be studied as models of the magnetic size effect [7] and as quasi-two-dimensional systems. The magnetic and phase transition properties of semi-infinite Ising systems have been investigated for many years.

The surface magnetism of these systems is very interesting [8–17]. It exhibits different types of phase transitions associated with the surface; if the ratio R, $(R = J_s/J)$, is greater than a critical value $R_c = (J_s/J)_{crit}$, the system may order on the surface before it orders in the bulk. As the temperature is lowered, the system undergoes two successive transitions, namely the *surface* and *bulk* phase transitions, whose critical temperatures are called the surface (T_c^S) and the bulk (T_c^B) critical temperatures respectively. On the other hand, if the ratio R is less than R_c, the whole system becomes ordered at the bulk transition temperature T_c^B.

Magnetic excitations in superlattices were considered in numerous papers (see, e.g., ref. [18] for a brief review). However, less attention has been paid to critical behavior, and in particular to critical temperatures in superlattices. Ma and Tsai [19] have studied the variation with the modulation wavelength of the Curie temperature for a Heisenberg magnetic superlattice. Their results agree qualitatively with experiments on Cu/Ni films [20]. Superlattice structures composed of alternating ferromagnetic and antiferromagnetic layers have been investigated by Hinchey and Mills [21,22], using a localized spin model. A sequence of spin-reorientation transitions are found to be different for superlattices with the antiferromagnetic component consisting of an even or odd number of spin layers.

Fishman et al. [23] have discussed, within the framework of the Ginzburg-Landau formalism, the static and dynamical properties of a periodic multilayer system formed of two different ferromagnetic materials. They have computed the transition temperature and the spin-wave spectrum. On the other hand, the Landau formalism of Camley and Tilley [24] has been applied to calculate the critical temperature in the same system [25]. Compared to ref. [23], the formalism of ref. [24] appears to be more general because it allows for a wider range of boundary conditions and includes the sign of exchange coupling across the interface.

For more complicated superlattices with arbitrary number of different layers in an elementary unit, Bernás [26] has derived some general dispersion equations for bulk and surface magnetic polaritons. These equations have been then applied to magnetostatic modes and to retarded wave propagation in the Voigt geometry [27].

On the other hand, with the developement of modern vacuum science and in

particular the epitaxial growth technique, it is possible to study experimentally the magnetic properties of low dimensional systems. For example, by depositing magnetic atoms on the top of non magnetic substrates, the thickness dependence of the critical temperature of ultrathin films of Gd on W(110) [28] and of Fe on Au(100) [29], has been measured.

Our aim in this paper is to study the phase diagrams of an Ising (spin-$\frac{1}{2}$) film in a transverse field within the framework of the effective field theory [30]. This technique is believed to give more exact results than those of the standard mean-field approximation. In section 2 we outline the formalism and derive the equations that determine the layer magnetizations, the average magnetizations and the critical temperature of the film as functions of temperature, exchange interactions, transverse fields and film thickness. The phase diagrams of the film are discussed in section 3. The last section is devoted to a brief conclusion.

2   Formalism

We consider a spin-$\frac{1}{2}$ Ising film of L layers on a simple cubic lattice with free surfaces parallel to the (001) plane, submitted to a transverse field. The Hamiltonian of the system is given by

$$H = -\sum_{<ij>} J_{ij}\sigma_i^z\sigma_j^z - \sum_i \Omega_i\sigma_i^x, \qquad (1)$$

where σ_i^z and σ_i^x respectively denote the z and x components of a quantum spin $\vec{\sigma}_i$ of magnitude $\sigma = \frac{1}{2}$ at site i, J_{ij} is the strength of the exchange interaction between the spins at nearest-neighbor sites i and j, and Ω_i represents the transverse field acting on the spin at site i. We assume $J_{ij} = J_{\mathrm{s}}$ if both spins belong to surface layers and $J_{ij} = J$ otherwise.

The statistical properties of the system are studied using an effective field theory whose the starting point is the generalized, but approximate, Callen [32] relation derived by Sá Barreto et al. [33] for the transverse Ising model. The longitudinal and transverse magnetizations of the spin at any site i are approximately given by (for details see Sá Barreto and Fittipaldi [34])

$$m_i^z = \langle\sigma_i^z\rangle = \frac{1}{2}\left\langle \frac{\sum_j J_{ij}\sigma_j^z}{[\Omega_i^2 + (\sum_j J_{ij}\sigma_j^z)^2]^{\frac{1}{2}}} \tanh\left(\frac{1}{2}\beta[\Omega_i^2 + (\sum_j J_{ij}\sigma_j^z)^2]^{\frac{1}{2}}\right)\right\rangle$$

$$= \left\langle f_z\left(\sum_j J_{ij}\sigma_j^z, \Omega_i\right)\right\rangle; \qquad (2)$$

$$m_j^x = \langle \sigma_i^x \rangle = \frac{1}{2} \left\langle \frac{\Omega_i}{[\Omega_i^2 + (\sum_j J_{ij}\sigma_j^z)^2]^{\frac{1}{2}}} \tanh\left(\frac{1}{2}\beta[\Omega_i^2 + (\sum_j J_{ij}\sigma_j^z)^2]^{\frac{1}{2}}\right) \right\rangle$$

$$= \left\langle f_x\left(\sum_j J_{ij}\sigma_j^z, \Omega_i\right) \right\rangle = \left\langle f_z\left(\Omega_i, \sum_j J_{ij}\sigma_j^z\right) \right\rangle ; \qquad (3)$$

where m_i^z and m_i^x are respectively the longitudinal and transverse magnetizations at site i, $\beta = 1/k_{\mathrm{B}}T$ (we take $k_{\mathrm{B}} = 1$ for simplicity), $\langle \ldots \rangle$ indicates the usual canonical ensemble thermal average for a given configuration, and the sum runs over all nearest neighbors of site i. We assume that the transverse field Ω depends only on the layer index, which we shall denote by n. Because of the translational symmetry parallel to the (001) plane, also the magnetizations only depend on n.

To perform thermal averaging on the right-hand side of eqs. (2,3), we follow the general approach described in ref. [30]. First of all, in the spirit of the effective field theory, multi-spin correlation functions are approximated by products of single spin averages. We then take advantage of the integral representation of the Dirac's delta distribution, in order to write eqs. (2,3) in the form

$$m_n^\alpha = \int \mathrm{d}\omega \, f_\alpha(\omega, \Omega_n) \frac{1}{2\pi} \int \mathrm{d}t \exp(\mathrm{i}\omega t) \prod_j \left\langle \exp(-\mathrm{i}t J_{ij}\sigma_j^z) \right\rangle, \qquad (4)$$

where $\alpha = z, x$ and

$$f_z(y, \Omega_n) = \frac{1}{2} \frac{y}{[y^2 + \Omega_n^2]^{\frac{1}{2}}} \tanh\left(\frac{1}{2}\beta[y^2 + \Omega_n^2]^{\frac{1}{2}}\right), \qquad (5)$$

$$f_x(y, \Omega_n) = \frac{1}{2} \frac{\Omega_n}{[y^2 + \Omega_n^2]^{\frac{1}{2}}} \tanh\left(\frac{1}{2}\beta[y^2 + \Omega_n^2]^{\frac{1}{2}}\right)$$

$$= f_z(\Omega_n, y). \qquad (6)$$

We now introduce the probability distribution of the spin variables (for details see Saber [30] and Tucker et al. [31]):

$$P(\sigma_n^z) = \frac{1}{2} \left[(1 - 2m_n^z)\delta\left(\sigma_n^z + \frac{1}{2}\right) + (1 + 2m_n^z)\delta\left(\sigma_n^z - \frac{1}{2}\right) \right]. \qquad (7)$$

Using this expression and eq. (4), we obtain the following set of equations for the longitudinal layer magnetizations:

$$m_1^z = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\mu_1=0}^{N_0} C_\mu^N C_{\mu_1}^{N_0} (1 - 2m_1^z)^\mu (1 + 2m_1^z)^{N-\mu}$$

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$$\times (1 - 2m_2^z)^{\mu_1}(1 + 2m_2^z)^{N_0 - \mu_1}$$

$$\times f_z\left(\frac{J}{2}\left[R(N - 2\mu) + (N_0 - 2\mu_1)\right], \Omega_1\right); \tag{8}$$

$$m_n^z = 2^{-N-2N_0} \sum_{\mu=0}^{N} \sum_{\mu_1=0}^{N_0} \sum_{\mu_2=0}^{N_0} C_\mu^N C_{\mu_1}^{N_0} C_{\mu_2}^{N_0}(1 - 2m_n^z)^{\mu}(1 + 2m_n^z)^{N-\mu}$$

$$\times (1 - 2m_{n-1}^z)^{\mu_1}(1 + 2m_{n-1}^z)^{N_0 - \mu_1}(1 - 2m_{n+1}^z)^{\mu_2}(1 + 2m_{n+1}^z)^{N_0 - \mu_2}$$

$$\times f_z\left(\frac{J}{2}\left[(N + 2N_0) - 2(\mu + \mu_1 + \mu_2)\right], \Omega_n\right),$$

$$\text{for } n = 2, 3, \ldots, L - 1; \tag{9}$$

$$m_L^z = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\mu_1=0}^{N_0} C_\mu^N C_{\mu_1}^{N_0}(1 - 2m_L^z)^{\mu}(1 + 2m_L^z)^{N-\mu}$$

$$\times (1 - 2m_{L-1}^z)^{\mu_1}(1 + 2m_{L-1}^z)^{N_0 - \mu_1}$$

$$\times f_z\left(\frac{J}{2}\left[R(N - 2\mu) + (N_0 - 2\mu_1)\right], \Omega_L\right). \tag{10}$$

In these equations we have introduced the notation $R = J_s/J$, N and N_0 are the number of nearest neighbors in the plane and between adjacent planes respectively, and C_k^l are the binomial coefficients, $C_k^l = l!/(k!(l - k)!)$. For the case of a simple cubic lattice, one has $N = 4$ and $N_0 = 1$.

The equations for the transverse magnetization for each layer are obtained by substituting the function f_x instead of f_z in the expression of the longitudinal magnetization. Since however $f_x(y, \Omega) = f_z(\Omega, y)$, this yields

$$m_n^x = m_n^z[f_z(y, \Omega_n) \longrightarrow f_x(y, \Omega_n)] = m_n^z[f_z(\Omega_n, y)]. \tag{11}$$

We have thus obtained a set of self consistent equations (8–11) for the layer longitudinal and transverse magnetizations m_n^z, m_n^x, that can be directly solved by numerical iteration. However, since we are interested in the calculation of the longitudinal order near the critical temperature, the usual argument that the layer longitudinal magnetizations m_n^z should tend to zero as the temperature approaches its critical value, allows us to consider only terms linear in m_n^z, because higher order terms tend to zero faster than m_n^z. Consequently, all terms of order higher than linear in eqs. (8–11) can be neglected. This leads to the following system of equations:

$$m_n^z = A_{n,n-1}m_{n-1}^z + A_{nn}m_n^z + A_{n,n+1}m_{n+1}^z, \tag{12}$$

which can be written as

$$\mathsf{A}\, m_n^z = m_n^z, \tag{13}$$

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where the matrix A is symmetric and tridiagonal with elements

$$A_{ij} = A_{ii}\delta_{ij} + A_{ij}(\delta_{i,j-1} + \delta_{i,j+1}). \tag{14}$$

For simplicity, we now assume that the transverse field acting on the system is uniform and equal to Ω. In this case, the only nonzero elements of the matrix A are given by

$$
\begin{aligned}
A_{11} = A_{LL} = {} & \frac{1}{2}\left\{ f_z\left(\frac{J}{2}(4R+1),\Omega\right) + f_z\left(\frac{J}{2}(4R-1),\Omega\right) \right. \\
& \left. + 2f_z\left(\frac{J}{2}(2R+1),\Omega\right) + 2f_z\left(\frac{J}{2}(2R-1),\Omega\right)\right\};
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
A_{12} = A_{L,L-1} = {} & \frac{1}{8}\left\{ f_z\left(\frac{J}{2}(4R+1),\Omega\right) - f_z\left(\frac{J}{2}(4R-1),\Omega\right) \right. \\
& \left. + 4f_z\left(\frac{J}{2}(2R+1),\Omega\right) - 4f_z\left(\frac{J}{2}(2R-1),\Omega\right) + 6f_z\left(\frac{J}{2},\Omega\right)\right\};
\end{aligned} \tag{16}
$$

$$A_{nn} = 4A_{n,n-1} = 4A_{n,n+1} = \frac{1}{4}\left\{ f_z(3J,\Omega) + 4f_z(2J,\Omega) + 5f_z(J,\Omega)\right\};$$
$$\text{for} \quad n = 2,3,\ldots,L-1. \tag{17}$$

The system of eqs. (12) is of the form

$$\mathsf{M}\, m_n^z = 0, \tag{18}$$

where the elements of the matrix M are given by

$$M_{ij} = (1 - A_{ii})\delta_{ij} - A_{ij}(\delta_{i,j-1} + \delta_{i,j+1}). \tag{19}$$

All the information about the critical temperature of the system is contained in eq. (18). So far we have not assigned explicite values to the coupling constants and the transverse field: the terms in matrix (18) are general ones.

In a general case, for arbitrary coupling constants, transverse field and film thickness, the evaluation of the critical temperature relies on numerical solution of the system of linear equations (18). These equations can be satisfied by nonzero magnetization vectors m_n^z only if

$$\text{Det}\,\mathsf{M} = 0, \tag{20}$$

where

$$\text{Det } \mathsf{M} = c \begin{vmatrix} a & -1 & & & & & \\ -1 & b & -1 & & & & \\ \ddots & \ddots & \ddots & \ddots & & & \\ & & -1 & b & -1 & & \\ & & & \ddots & \ddots & \ddots & \ddots \\ & & & & -1 & b & -1 \\ & & & & & -1 & a \end{vmatrix}_L . \tag{21}$$

The parameters a, b and c that appear in equation (21) take into account the boundary conditions and represent the different propensities to order of the surfaces and of the bulk. They are given by

$$a = \frac{1 - A_{11}}{A_{12}}; \tag{22}$$

$$b = \frac{1 - A_{nn}}{A_{n,n-1}} = \frac{1 - A_{nn}}{A_{n,n+1}} = \frac{1 - A_{nn}}{A_{nn}/4}; \qquad \text{for} \quad n = 2, 3, \dots, L-1; \tag{23}$$

$$c = \left(\frac{1}{A_{12}}\right)^2 \left(\frac{1}{A_{nn}/4}\right)^{2(L-2)} . \tag{24}$$

In general, equation (20) can be satisfied for L different values of the critical temperature T_{c}/J from which we choose the one corresponding to the highest possible transition temperature (cfr. the discussion in refs. [34,35]). This value of T_{c}/J corresponds to a solution where m_1^z, m_2^z,..., m_L^z are all positive, which is compatible with a ferromagnetic longitudinal ordering. The other solutions correspond in principle to other types of ordering that usually do not occur here [34].

The reduction and rearrangement of the determinant of eq. (21) leads to the result [36–38]

$$\text{Det } \mathsf{M} = c[(ab-1)^2 D_{L-4}b - 2a(ab-1)D_{L-5}b + a^2 D_{L-6}b], \tag{25}$$

where $D_L(x)$ is the determinant

$$D_L(x) = \begin{vmatrix} x & -1 & & & & & \\ -1 & x & -1 & & & & \\ \ddots & \ddots & \ddots & \ddots & & & \\ & & -1 & x & -1 & & \\ & & & \ddots & \ddots & \ddots & \ddots \\ & & & & -1 & x & -1 \\ & & & & & -1 & x \end{vmatrix}_L , \qquad (26)$$

whose value is given by

$$\begin{aligned} D_L(x) &= (x^2 - 4)^{-\frac{1}{2}} 2^{-(L+1)} \\ &\quad \times \left\{ \left[x + \sqrt{x^2 - 4} \right]^{L+1} - \left[x - \sqrt{x^2 - 4} \right]^{L+1} \right\}, \quad \text{for } x^2 > 4; \quad (27) \\ &= \sin[(L+1)k]/\sin k, \ \text{with } k = \cos^{-1}(x/2), \qquad \text{for } x^2 \le 4. \quad (28) \end{aligned}$$

From now on, we take J as the unit of energy in our numerical calculations, and we measure length in units of the lattice constant.

3   Phase diagrams

From eqs. (20) and (25), we can obtain the phase diagrams of the film. The results show that there can be two phases, a film ferromagnetic phase (F), in which the longitudinal magnetization ($\overline{m}_z = \frac{1}{L}\sum_{n=1}^{L} m_n^z$) is different from zero, and a film paramagnetic phase (P), in which $\overline{m}_z = 0$. In addition, if the number of layers in the film, L, is very large ($L \to \infty$), the film should practically behave as a semi-infinite Ising system. It is well known that, if the ratio $R = J_s/J$ of surface to bulk exchange interactions in a semi-infinite Ising system, is greater than a critical value $R_c = (J_s/J)_{\text{crit}}$, there appear two different transitions: the *surface* transition and the *bulk* transition. The critical temperatures related to them are respectively called the *surface* critical temperature T_c^{S} and the *bulk* critical temperature T_c^{B}. To obtain the bulk and surface critical temperatures of the semi-infinite Ising system, we follow the approach due to Binder and Hohenberg [39]. Using eqs. (22–24), the system of linear equations (13) yields

$$a m_1^z - m_2^z = 0; \qquad (29)$$

$$-m_1^z + bm_2^z - m_3^z = 0; \tag{30}$$
$$-m_{n-1}^z + bm_n^z - m_{n+1}^z = 0, \qquad \text{for } n \geq 3. \tag{31}$$

According to Binder and Hohenberg [39], let us assume that $m_{n+1}^z = \gamma \, m_n^z$ for $n \geq 3$, e.g., the layer magnetization m_n^z of each layer, with n larger than 2, decreases exponentially in the bulk. The equations (29) and (30) then yield the following secular equation:

$$M_s \begin{pmatrix} m_1^z \\ m_2^z \end{pmatrix} = \begin{pmatrix} a & -1 \\ -1 & b - \gamma \end{pmatrix} \begin{pmatrix} m_1^z \\ m_2^z \end{pmatrix} = 0, \tag{32}$$

where, from eq. (31), the parameter γ is given by

$$\gamma = \frac{1}{2} \left(b - \sqrt{b^2 - 4} \right). \tag{33}$$

Thus, the surface critical temperature T_c^S/J can be derived from the condition $\text{Det } M_s = 0$, namely

$$a \, (b - \gamma) - 1 = 0. \tag{34}$$

We can now study numerically the physical properties of the surface and the bulk of the semi-infinite Ising system. Here it is worth noting that in our treatment the bulk transition temperature can be determined by letting $m_n^z = m_{n-1}^z = m_{n+1}^z = m^z$ in eq. (31), i.e.,

$$b - 2 = 0. \tag{35}$$

This yields

$$f_z(3J, \Omega) + 4f_z(2J, \Omega) + 5f_z(J, \Omega) = 8/3. \tag{36}$$

At $T_c^B/J = 0$, eq. (36) yields the bulk critical transverse field value: $\Omega_c^B/J = 2.3529$. On the other hand, for the special case of the Ising model in the absence of the transverse field ($\Omega = 0$), eq. (36) reduces to

$$\tanh(3\beta J) + 4\tanh(2\beta J) + 5\tanh(\beta J) = 16/3, \tag{37}$$

which is the Zernike [40] equation for the simple cubic lattice. The transition temperature is then determined as $T_c^B/J = 1.2683$.

A useful expression for determining the critical value $R_c = (J_s/J)_{\text{crit}}$ is therefore given by the simultaneous solution of the equations (33) and (34). The
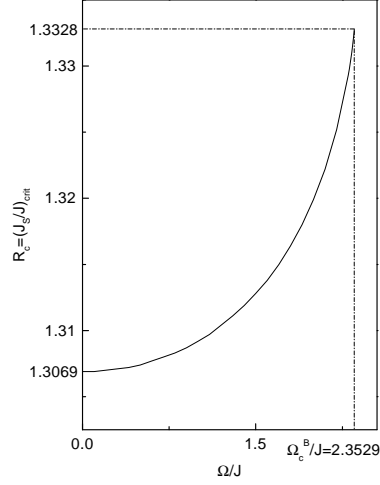
Fig. 1. The variation of $R_c = (J_s/J)_{crit}$ as a function of the transverse field Ω/J.

variation of R_c as a function of the transverse field is shown in Fig. 1. It shows that R_c increases with the increase of the strength of the transverse field from its minimal value $R_c^{min} = 1.3069$ for $\Omega/J = 0$ and reaches its maximal value $R_c^{max} = 1.3328$ at the bulk critical transverse field $\Omega_c^B/J = 2.3529$.

We now calculate the $(T_c/J, R = J_s/J)$ phase diagrams for different values of the transverse field and of the number of layers: typical results are shown in Fig. 2. Our phase diagrams are qualitatively different from the corresponding diagrams for the semi-infinite ferromagnet. The main difference is that we get in the film only one well defined critical temperature T_c/J, instead of the two critical temperatures T_c^B/J and T_c^S/J. This temperature depends on the film thickness. According to these results we must give a new definition of R_c. In semi-infinite systems R_c was defined as the value of the parameter R above which the two critical temperatures T_c^B/J and T_c^S/J exist. However, according to Fig. 2 the parameter R_c can now be defined as that particular value of R at which the critical temperature does not depend on film thickness (the cross-over point in Fig. 2). As it has been stated the numerical values of R_c and related T_c/J parameters are exactly the same as those found for the semi-infinite system. Furthermore, according to the definition of R_c, it can be expected that the cross-over point in Fig. 2 should also define the critical temperature of the three-dimensional infinite bulk system, where the surfaces and the R parameter are of no importance. Fig. 2, where the bulk and the surface critical temperatures of the corresponding semi-infinite system are represented respectively by the dashed and dotted lines also shows that this is really the case.
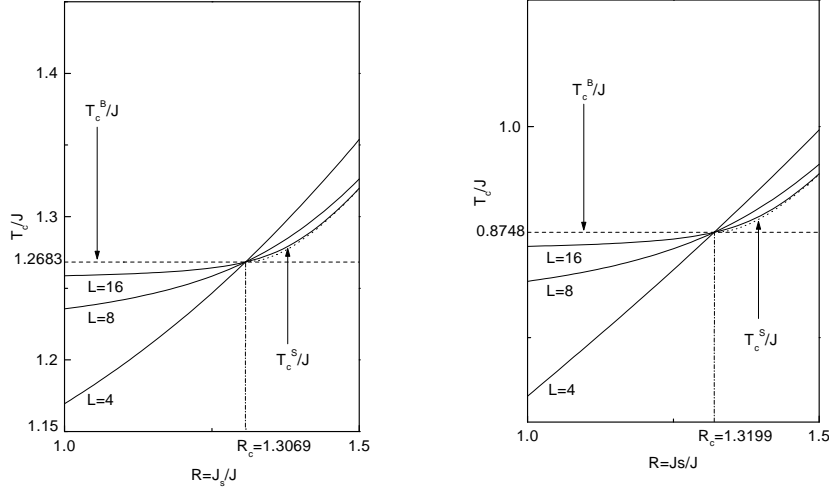
Fig. 2. The phase diagram in the $(T_c/J, R = J_s/J)$ plane. (a) $\Omega/J = 0$; (b) $\Omega/J = 2$. The dashed line is the bulk critical temperature T_c^B/J and the dotted line is the surface critical temperature T_c^S/J of the corresponding semi-infinite Ising system.

From Fig. 2a, which corresponds to the case when there is no transverse field acting on the system, we find that the value of R_c corresponding to the cross-over point is equal to 1.3069, which is equal to the value reported by Wiatrowski et al. [42] and by Sarmento and Tucker [43]. For $R < R_c$, the critical temperature T_c/J of the film is smaller than the bulk critical temperature T_c^B/J, and T_c/J increases with the increase of L, approaching the bulk critical temperature $T_c^B/J = 1.2683$ asymptotically as the number of layers becomes large. When $R = R_c$, the critical temperature of the film T_c/J is independent of L, and equal to T_c^B/J. On the other hand, for $R > R_c$ the critical temperature of the film T_c/J is larger than both the bulk T_c^B/J and the surface T_c^S/J critical temperatures of the corresponding semi-infinite system. The larger L, the lower T_c/J, and, when the number of layers L becomes large, T_c^B approaches asymptotically T_c^S/J.

We now study the influence of the surface exchange interactions in the presence of a transverse field. Fig. 2b shows that a transverse field acting on the system increases the critical value of R_c. For example, with a transverse field $\Omega/J = 2$, the critical value of R_c is shifted from 1.3069 to 1.3199. At the same time of course for a given ratio of the exchange interactions $R = J_s/J$, the critical temperatures of the film and of the corresponding semi-infinite system are reduced.

In Figs. 3, we show the phase diagrams of the film in the $(T_c/J, \Omega/J)$-plane with different values of the thickness L, and the critical temperatures of the
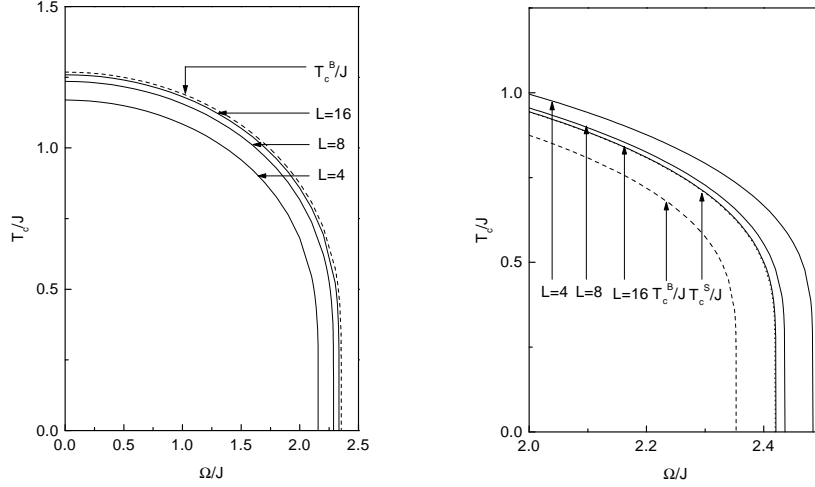
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Fig. 3. Phase diagram in the $(T_c/J, \Omega/J)$ plane: (a) $R = 1$; (b) $R = 1.5$. The dashed line is the bulk critical temperature T_c^B/J and the dotted line is the surface critical temperature T_c^S/J of the corresponding semi-infinite Ising system.

semi-infinite Ising system for different values of the parameter R. The presence of a transverse field, of course, reduces the critical temperatures of the film and of the semi-infinite system. We find that the $(T_c/J, \Omega/J)$ curve for a given value of R intercepts the (Ω/J)-axis at a critical value Ω_c/J of the transverse field. When $\Omega/J > \Omega_c/J$, there cannot be a ferromagnetic phase at any temperature. Fig. 3a, which corresponds to $R = 1$ (smaller than R_c^{min}), shows that, for any finite value of the film thickness L, the film critical temperature T_c/J is smaller than the bulk critical temperature T_c^B/J and increases as L increases, approaching T_c^B/J for large values of L. Fig. 3b corresponds to $R = 1.5 > R_c^{min}$, and shows that the film critical temperature T_c/J is larger than both the bulk critical temperature T_c^B/J and the surface critical temperature T_c^S/J of the corresponding semi-infinite system. T_c/J decreases with the increase of L, approaching T_c^S/J for large values of L.

We show in Fig. 4 the thickness dependence of the critical temperature of the film for different values of R and Ω/J. Fig. 4a corresponds to the case when the transverse field vanishes. It shows that for any value of R below the critical value $R_c = R_c^{min} = 1.3069$, the critical temperature of the film increases with L and approaches the bulk critical temperature $T_c^B/J = 1.2683$ asymptotically as the number of layers becomes large. On the other hand, for $R > R_c$, the critical temperature of the film becomes smaller as the number of layers increases, and approaches asymptotically, for large values of L, the surface critical temperature T_c^S/J which depends on R. In Fig. 4b, we show
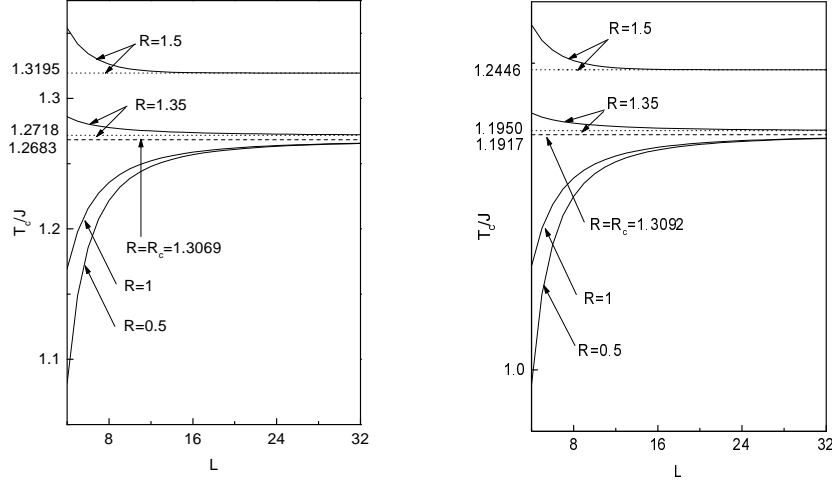
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Fig. 4. Thickness dependence of the critical temperature of the film for (a) $\Omega/J = 0$, and (b) $\Omega/J = 1$. The dashed line is the bulk critical temperature T_c^B/J and the dotted line is the surface critical temperature T_c^S/J of the corresponding semi-infinite Ising system.

the thickness dependence of the critical temperature of the film when there is a transverse field acting on the system. We consider the case when $\Omega/J = 1$ for several values of R. We see that Fig. 4b exhibits the same qualitative behavior as Fig. 4a, except that the transverse field reduces the critical temperatures.

Finally Fig. 5 shows the variation of the critical transverse field Ω_c/J as a function of the thickness of the film L for several values of R. The dashed and dotted lines correspond respectively to the bulk and surface critical transverse fields of the corresponding semi-infinite Ising system. For $R \leq R_c$ the critical transverse field of the film, Ω_c/J, is smaller than the bulk critical transverse field Ω_c^B/J, and increases with the increase of L, approaching Ω_c^B/J for large values of L. For $R > R_c$, Ω_c/J is larger both than Ω_c^B/J and Ω_c^S/J, and decreases as L increases, approaching Ω_c^S/J for large values of L.

4   Conclusion

We have studied, within the effective field theory, the phase diagram of the transverse spin-$\frac{1}{2}$ Ising film, where the exchange interactions between spins on the surfaces are different from those in the bulk. We have investigated the effects of the surface to bulk exchange interaction ratio, of the strength
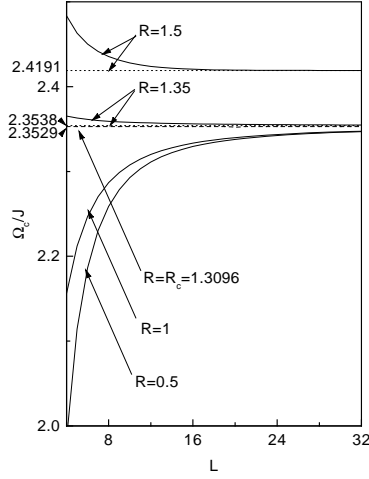
Fig. 5. The variation of the critical transverse field of the film Ω_c/J at which the critical temperature of the film T_c/J becomes zero as function of the number of layers L. The dashed line and the dotted line correspond respectively to the bulk and surface critical fields of the semi-infinite Ising system.

of the transverse field and of the film thickness on the phase diagram. The results show that, in the film, there is only one critical temperature T_c/J which depends on L, R and Ω/J. We have identified a critical value R_c of the parameter R, such that when $R = R_c$, T_c/J is independent of L; for $R \leq R_c$, T_c/J is smaller than T_c^B/J and for $R > R_c$, T_c/J is greater both than T_c^B/J and T_c^S/J. When L becomes very large, then for $R \leq R_c$ ($R > R_c$), T_c/J approaches T_c^B/J (T_c^S/J) of the corresponding semi-infinite Ising system.

Acknowledgements

References

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[1] For a review, see: R. E. Camley and R. L. Stamps, *J. Phys.: Condens. Matter* **5** (1993) 3727.

[2] G. Shaulov and Yu Seidov, *J. Magn. Magn. Mater.* **140–144** (1995) 527.

[3] T. Balcerzak and J. W. Tucker, *J. Magn. Magn. Mater.* **140–144** (1995) 653.

[4] T. Hai, Z. V. Li, D. L. Lin and Th. F. George, *J. Magn. Magn. Mater.* **97** (1991) 227.

[5] A. R. Ferchimin, *IEEE Trans. Magn.* **MAG-18** (1982) 714.

[6] J. Mielnicki, T. Balcerzak and G. Wiatrowski, *J. Magn. Magn. Mater.* **65** (1987) 27.

[7] M. N. Barber, in: C. Domb and J. L. Lebowitz (eds.), *Phase Transitions and Critical Phenomena*, Vol. **8** (Academic Press, New York, 1983).

[8] E. F. Sarmento and T. Kaneyoshi, *J. Phys. C: Solid State Phys.* **21** (1988) 3993.

[9] Qing Jiang and Li Zhen-Ya, *Phys. Rev. B* **43** (1991) 6198.

[10] D. L. Mills, *Phys. Rev. B* **3** (1971) 3887.

[11] K. Binder and P. C. Hohenberg, *Phys. Rev. B* **9** (1974) 2194.

[12] K. Binder and D. P. Landau, *Phys. Rev. Lett.* **52** (1984) 318.

[13] T. Kaneyoshi, *Introduction to Surface Magnetism* (Chem. Rubber Co., Boca Raton, 1991).

[14] A. Benyoussef, N. Boccara and M. Saber, *J. Phys. C: Solid State Phys.* **18** (1985) 4275.

[15] A. Benyoussef, N. Boccara and M. Saber, *J. Phys. C: Solid State Phys.* **19** (1986) 1983.

[16] M. Saber, *J. Phys. C: Solid State Phys.* **20** (1987) 2749.

[17] E. F. Sarmento, M. Saber and T. Kaneyoshi, *Phys. Stat. Sol. b* **171** (1992) 247.

[18] J. A. Morkowski and A. Szajek, *J. Magn. Magn. Mater.* **71** (1988) 299.

[19] H. R. Ma and C. H. Tasi, *Solid State Commun.* bf 55 (1985) 499.

[20] J. Q. Zheng, J. B. Ketterson, C. M. Falco and I. K. Schuller, *J. Appl. Phys.* **53** (1982) 3150.

[21] L. L. Hinchey and D. L. Mills, *Phys. Rev. B* **33** (1986) 3329.

[22] L. L. Hinchey and D. L. Mills, *Phys. Rev. B* **34** (1986) 1689.

[23] F. Fishman, F. Schwable and D. Schwenk, *Phys. Lett. A* **121** (1987) 192. (1987).

[24] R. E. Camley and D. R. Tilley, *Phys. Rev. B* **37** (1988) 3413.

[25] D. R. Tilley, *Solid State Commun.* **65** (1988) 657.

[26] J. Bernás, *J. Phys. C: Solid State Phys.* **21** (1988) 1021; *ibid.* 4097.

[27] J. Bernás, J. Phys.: Condens. Matter **2** (1990) 7173.

[28] M. Farle and K. Baberschke, *Phys. Rev. Lett.* **58** (1987) 511.

[29] W. Dürr, M. Taborelli, O. Paul, R. Germar, W. Gudat, D. Pescia and M. Landot, *Phys. Rev. Lett.* **62** (1989) 206.

[30] M. Saber, to appear in Chinese Journal of Physics.

[31] J. W. Tucker, M. Saber and L. Peliti, Physica A **206** (1994) 497.

[32] H. B. Callen, *Phys. Lett.* **4** (1963) 161.

[33] F. C. Sá Barreto, I. P. Fittipaldi and B. Zeks, *Ferroelectrics* **39** (1981) 1103.

[34] F. C. Sá Barreto and I. P. Fittipaldi, *Physica A* **129** (1985) 360.

[35] A. R. Ferchmin and W. Maciejewski, *J. Phys. C: Solid State Phys.* **12** (1979) 4311.

[36] A. R. Ferchmin and S. Krompiewski, *J. Phys. C: Solid State Phys.* **8** (1975) 1901. (1975).

[37] D. E. Rutherford, *Proc. R. Soc. A* **62** (1947) 229.

[38] H. Puskarski, *IEEE Trans. Magn.* **MAG-9** (1973) 22.

[39] K.Binder and P. C. Hohenberg, *Phys. Rev. B* **9** (1974) 2149.

[40] F. Zernike, *Physica* **7** (1940) 565.

[41] G. Wiatrowski, J. Mielnicki and T. Balcerzak, *Phys. Stat. Sol. b* **164** (1991) 299.

[42] E. F. Sarmento and J. W. Tucker, *J. Magn. Magn. Mater.* **118** (1993) 133.